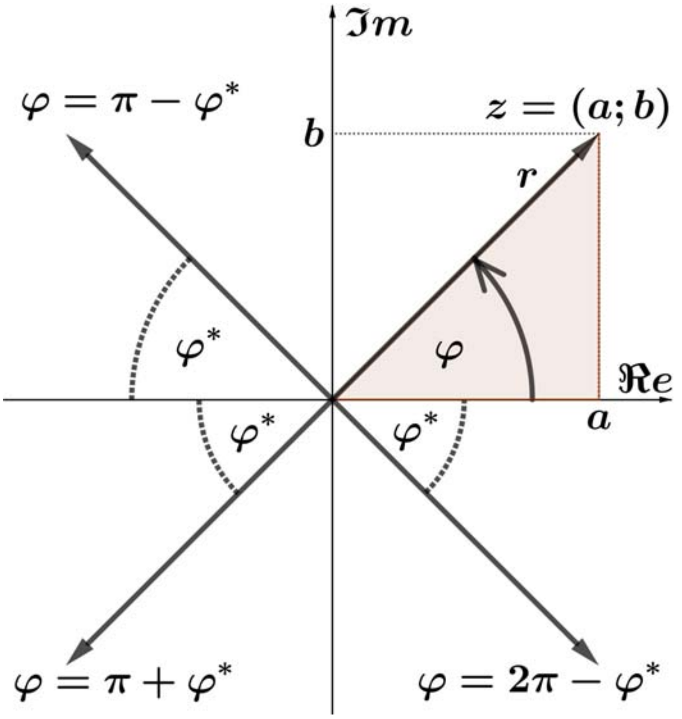
## Komplex számok halmaza: $\mathbb{C}$

$$\mathbb{C}=\left\{z \mid z=(a;b)\in\mathbb{R}^2\right\},\quad \Re e(z)=a,\quad \Im m(z)=b$$

$$1=(1;0),\quad i=(0;1),\quad i^2=-1$$

| | |
|---|---|
| Algebrai alak | $z=a+bi$ |
| Trigonometrikus alak | $z=r\cdot(\cos\varphi+i\sin\varphi)$ |
| Exponenciális alak | $z=r\cdot e^{\varphi i}$ |

$$r=\lvert z\rvert=\sqrt{a^2+b^2},\quad \varphi\in[0;2\pi[,\quad \varphi=\frac{\varphi^0}{180^0}\cdot\pi$$




Komplex konjugált: $\overline{z}=(a;-b)$

| | |
|---|---|
| Algebrai alak | $\overline{z}=a-bi$ |
| Trigonometrikus alak | $\overline{z}=r\cdot(\cos(-\varphi)+i\sin(-\varphi))$ |
| Exponenciális alak | $\overline{z}=r\cdot e^{-\varphi i}$ |

Műveletek komplex számokkal

| | |
|---|---|
| $z_1=$ | $a_1+b_1i=r_1\cdot(\cos\varphi_1+i\sin\varphi_1)=r_1\cdot e^{\varphi_1 i}$ |
| $z_2=$ | $a_2+b_2i=r_2\cdot(\cos\varphi_2+i\sin\varphi_2)=r_2\cdot e^{\varphi_2 i}$ |
| $z_1+z_2=$ | $(a_1+a_2)+(b_1+b_2)i$ |
| $z_1-z_2=$ | $(a_1-a_2)+(b_1-b_2)i$ |
| $z_1\cdot z_2=$ | $(a_1\cdot a_2-b_1\cdot b_2)+(a_1\cdot b_2+a_2\cdot b_1)i$ |
| | $r_1\cdot r_2\cdot(\cos(\varphi_1+\varphi_2)+i\sin(\varphi_1+\varphi_2))$ |
| | $r_1\cdot r_2\cdot e^{(\varphi_1+\varphi_2)i}$ |
| $\frac{z_1}{z_2}=$ | $\frac{z_1}{z_2}\cdot\frac{\overline{z_2}}{\overline{z_2}}$ |
| | $\frac{r_1}{r_2}\cdot(\cos(\varphi_1-\varphi_2)+i\sin(\varphi_1-\varphi_2))$ |
| | $\frac{r_1}{r_2}\cdot e^{(\varphi_1-\varphi_2)i}$ |

| | |
|---|---|
| $i^n=$ | $\begin{cases}1, & ha\quad n=4\cdot k\\ i, & ha\quad n=4\cdot k+1\\ -1, & ha\quad n=4\cdot k+2\\ -i, & ha\quad n=4\cdot k+3\end{cases}$ $ahol\quad k\in\mathbb{N}$ |
| $z^n=$ | $(a+bi)^n=\sum_{k=0}^{n}\binom{n}{k}\cdot a^{n-k}\cdot b^k\cdot i^k,\quad (n\in\mathbb{N})$ |
| | $r^n\cdot(\cos(n\cdot\varphi)+i\sin(n\cdot\varphi)),\quad (n\in\mathbb{N})$ |
| | $r^n\cdot e^{n\cdot\varphi i},\quad (n\in\mathbb{N})$ |
| $z^{-n}=$ | $\frac{1}{z^n},\quad (n\in\mathbb{N}, z\neq 0)$ |
| $\sqrt[n]{z}=$ | $\sqrt[n]{r}\cdot\left(\cos\left(\frac{\varphi}{n}+k\cdot\frac{2\pi}{n}\right)+i\sin\left(\frac{\varphi}{n}+k\cdot\frac{2\pi}{n}\right)\right),$ $ahol\quad k=0,1,2,\ldots,n-1$ |
| | $\sqrt[n]{r}\cdot e^{\left(\frac{\varphi}{n}+k\cdot\frac{2\pi}{n}\right)i},\quad ahol\quad k=0,1,2,\ldots,n-1$ |
| $\sqrt[n]{1}=$ | $\varepsilon_{n,k}=\cos\left(k\cdot\frac{2\pi}{n}\right)+i\sin\left(k\cdot\frac{2\pi}{n}\right)=e^{k\cdot\frac{2\pi}{n}i},$ $ahol\quad k=0,1,2,\ldots,,n-1,\quad \sum_{k=0}^{n-1}\varepsilon_{n,k}=0$ |
| $\sqrt{z}=$ | $\begin{cases}\sqrt{\frac{a+\sqrt{a^2+b^2}}{2}}+i\sqrt{\frac{-a+\sqrt{a^2+b^2}}{2}}\\ -\sqrt{\frac{a+\sqrt{a^2+b^2}}{2}}-i\sqrt{\frac{-a+\sqrt{a^2+b^2}}{2}}\end{cases}$ |
| $Lnz=$ | $\ln r+i(\varphi+k\cdot 2\pi),\quad r>0,\quad k\in\mathbb{Z}$ |
| $\overline{z_1+z_2}=$ | $\overline{z_1}+\overline{z_2}$ |
| $\overline{z_1-z_2}=$ | $\overline{z_1}-\overline{z_2}$ |
| $\overline{z_1\cdot z_2}=$ | $\overline{z_1}\cdot\overline{z_2}$ |
| $z\cdot\overline{z}=$ | $r^2$ |
| $\overline{\left(\frac{z_1}{z_2}\right)}=$ | $\frac{\overline{z_1}}{\overline{z_2}}$ |
| $\overline{z^n}=$ | $\overline{z}^n,\quad (n\in\mathbb{N})$ |